$P(B) \quad P(B)$

$$\frac{4}{15} \cdot \frac{3}{14}$$

$P(R) \cdot P(O)$

$$\frac{2}{15} \cdot \frac{3}{14} =$$

AP Stat- Ch. 14 & 15

Intro Vocab:

Experimental Probability-

what DOES happen

Ex: If I toss a coin 30 times, and get 12 heads, what's the **experimental** prob. of getting heads? $12/30 =$

---

parameter

Theoretical Probability-

What SHOULD happen

Ex: Using the same coin tossing situation above, what's the **theoretical** prob. of getting heads?

$P(H) = 50\%$

Sample Space-

all possible outcomes

Example: What is the sample space for the spinner experiment?

Red, Green, Blue, Yellow

What about the rolling 2 dice experiment?

2 — 12

$\frac{\bigcirc}{52}$

Probability Notation:

- A, B, C, etc. = events

  P(J) = P(RA)

- P(A) = prob. of event A happening

- S = sample space

Probability Rules

- Let A and B be events
- Let S = sample space
- Let A^c = the complement of event A

"opposite"
"not"

The 3 Probability Rules

(1) $0 \le P(A) \le 1$

(2) $P(S) = 1$

(3) $P(A^c) = 1 - P(A)$

Example 1: If the probability of hitting a homerun is 30%, what's the probability of not hitting a homerun? P(H) = 30% = 0.3 P(H^c) = 70% = 0.70

Example 2: If there are only 8 different blood types, fill in the chart below:

AB+ = 0.06

P(Duke) = 0.3 P(NCState) = 0.1

P(UNC) = 0.6 P(UVA) = 0.1

Example 3: Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

Venn Diagrams

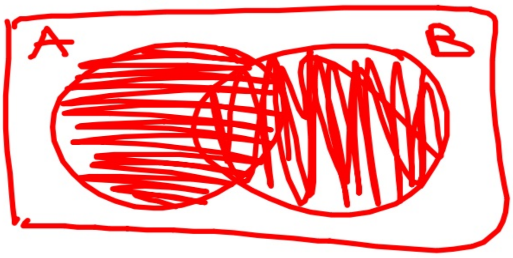
UNION

- **Meaning:** or, addition

- Symbol: $\cup$

Example 1:

$A \cup B$



Example 2: Set A = {2, 4, 6, 8, 10, 12}

Set B = {1, 2, 3, 4, 5, 6, 7}

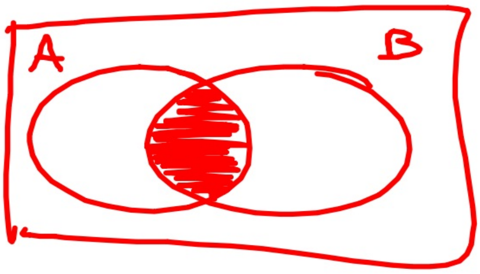
{1, 2, 3, 4, 5, 6, 7, 8, 10, 12}

A U B = A or B = { }

Intersection:

- Meaning: and, overlap
- Symbol: $\cap$

Example 1:



Example 2: Set A = {2, 4, 6, 8, 10, 12}

Set B = {1, 2, 3, 4, 5, 6, 7}

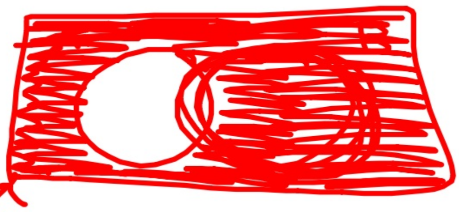
$A \cap B$ = A and B = { 2, 4, 6 }

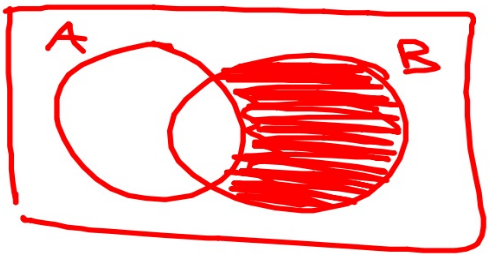
Complement (of A)

- Meaning: "not"

- Symbol: A^c A'

- Example 1: Shade A^c Shade $A^c \cap B$





Sample Space

- Example 2: Set A = {2, 4, 6, 8, 10, 12}

A^c ~~S~~ = {1, ~~2~~, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, 9, ~~10~~, 11, ~~12~~, 13, 14, 15} = ~~sample space~~

A^c = { }

More Probability Rules

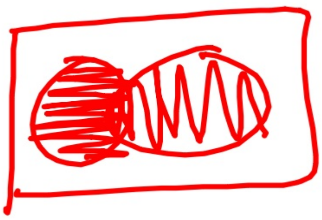
UNIONS:

Example: For a deck of cards….. What is the probability of picking a card that is a red card OR a face card?

P(Red or Face)

26 + 12 =

− 6 =



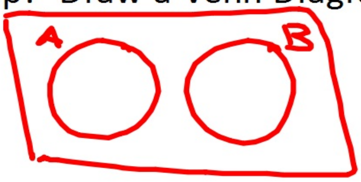
General Rule:

* P(A U B) = P(A) + P(B) − P(A ∩ B)
* Why do we subtract P(A∩B)?

* double count *

Special Case:

What if A and B don't overlap? Draw a Venn Diagram that illustrates this below:



So, P(A∩B) = 0

This is called **Disjoint**

Disjoint (or mutually exclusive)= Two events are disjoint if …

they have no outcomes in common

Example: Back to the deck of cards…. What is the probability of picking a red card OR a club?

$P(R \text{ or Club})$

$P(R) + P(\text{Club}) - \cancel{P(R \cap \text{Club})}$ 0

So our rule for **unions** for **disjoint** events then becomes:

- P(A U B) = $P(A) + P(B)$

Going back to the examples from before…

Ex #4: There are only 8 different blood types, given in the chart below:

Are these events (the blood types) disjoint?

What is the probability of being either Type A+ or B-?

What is the probability of being either Type O- or O+?

What is the probability of being either Type AB+ or A+?

Ex #5: We are picking one card out of a standard 52-card deck (no jokers).

The events are the different cards we can pick.

What is the probability of picking a diamond?

$P(D) = 1/4 = 13/52$

What is the probability of picking a 3?

$P(3) = 4/52$

What is the probability of picking a diamond and a 3?

$P(D \cap 3) = 1/52$

So, what is the probability of picking a diamond OR a 3?

$P(D \cup 3) = 13/52 + 4/52 - 1/52 = 16/52$

What is the probability of picking a black card?

What is the probability of picking a Jack?

What is the probability of picking a black card or a Jack?

Ex #6: The probability of event G is 0.25 and event K is 0.34. $P(G \cap K) = 0.1$.

What is $P(G \cup K)$?

$P(G) = 0.25$

$P(K) = 0.34$

$P(G \cap K) = 0.1$

$$P(G \cup K) = 0.25 + 0.34 - 0.1$$
$$= 0.49$$

Probability Rules (cont'd)

INTERSECTIONS:

EX #7: I roll 2, regular 6-sided dice and add together the faces..... the probabilities are:

2 = 1/36	7 = 6/36
3 = 2/36	8 = 5/36
4 = 3/36	9 = 4/36
5 = 4/36	10 = 3/36
6 = 5/36	11 = 2/36
	12 = 1/36

(a) What is the probability of rolling a 6? What is the probability of rolling a 10?

$5/36$ $3/36$

(b) What is the probability of rolling a 6 and then a 10?

$P(6)P(10)$ $\left(5/36\right)\left(3/36\right) =$

(c) What is the probability of rolling a 3 and then a 7?

$P(3)\ P(7) =$

EX #8: Let's go back to the worksheet… I have a small bag of M&M's, where there are 4 blue, 3 orange, 2 red, and 6 green. I pick one out, look at the color, and then replace it. Find the following probabilities:

(a) Picking a red and then a green

$P(R)\ P(G) = \frac{2}{15} \cdot \frac{6}{15}$

(b) Picking a blue and then another blue

$P(B) \cdot P(B) = \frac{4}{15} \cdot \frac{4}{15}$

(c) Picking a red and then an orange

Question: Did the first event happening ~~affect~~ the second event happening?

1st doesn't affect 2nd

This is called… INDEPENDENT

If A and B are independent, then $P(A \cap B)$ = $P(A) \cdot P(B)$

$P(A \cap B \cap C)$

EX #8: Back to the M&M's … I have a small bag of M&M's, where there are 4 blue, 3 orange, 2 red, and 6 green. I pick one out, look at the color, and then EAT IT before I pick another. Find the following probabilities:

$P(G|R)$

(a) Picking a red and then a green

$P(R) \cdot \cancel{P(G)} = \frac{2}{15} \cdot \frac{6}{14}$

(b) Picking a blue and then another blue

$P(B) \cdot P(B|B) = \frac{4}{15} \cdot \frac{3}{14}$

(c) Picking a red and then an orange

$P(R) \cdot P(O|R) = \frac{2}{15} \cdot \frac{3}{14}$

What is different??

1st event affects 2nd event

VOCAB: Conditional Probabilities:

$$P(B|A)$$

A = 1st event that happened

B = 2nd event that happened

* not independent = generic

INTERSECTIONS: **P(A and B) = P(A $\cap$ B) =** $P(A) \cdot P(B|A)$

REARRANGE: **P(B|A) =** $\dfrac{P(A \cap B)}{P(A)}$

EX #9: P(J) = 0.23 and P(B) = 0.67 and P(J|B) = 0.15. What is $P(J \cap B)$?

$$P(J \cap B) = P(B) \cdot P(J|B)$$

$$(0.67)(0.15)$$

EX #10: P(A) = 0.45 and P(C) = 0.39 and $P(A \cap C)$ = 0.22. What is P(A|C)?

$$P(A \cap C) = P(C \cap A)$$

What is P(C|A)?

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{0.22}{0.39} = 0.564$$

$$P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{0.22}{0.45} = 0.489$$

EX #11: Look at the following table about grade level and favorite type of pet and answer the probability questions:

	Frosh	Soph	Junior	Senior	
Dog	14	18	22	16	70
Cat	8	11	13	15	47
Other	12	14	10	9	45
	34	43	45	40	162

(a) If someone is a sophomore, what is the probability they like Dogs the most?

$P(D|S) = 18/43$ $= \frac{P(D \cap So)}{P(So)}$

(b) Given that someone likes Cats the most, what is the probability that they are a junior?

$P(J|C) = 13/47$

(c) We pick a freshman at random. What is the probability that they like other the most?

$P(O|F) = 12/34$

Thinking about conditional probabilities… **Does P(B|A) = P(A|B)?**

Think about our M&M example. I have a small bag of M&M's, where there are 4 blue, 3 orange, 2 red, and 6 green. I pick one out, look at the color, and then EAT IT before I pick another.

- What is the probability of picking red given that you picked blue?

$P(R|B)$

- What is the probability of picking blue given that you picked red?

$P(B|R)$

So… $P(A|B) \neq P(B|A)$

Another concept from Ch. 14:

The Law of Large Numbers:

In the long run, the P(A) in an experiment gets closer to the true theoretical P(A) as the number of trials increase.

exp. prob $\longrightarrow$ theoretical prob

as $n \uparrow$

TRY THE WORKSHEET!

Answers:

1) (a) $P(A) + P(B) - P(A \text{ and } B) = 0.57$

   (b) $P(A \text{ and } B)/P(A) = 0.3846$

   (c) NO! $P(A \text{ and } B) \neq 0$

   (d) NO! $P(B|A) \neq P(B)$

2) $P(G)*P(M) = 0.1386$

3) (a) $P(J|W) * P(W) = 0.12 = P(W \cap J)$

   (b) $P(W) + P(J) - P(W \text{ and } J) = 0.82 = P(W \cup J)$

4) (a) 0.83
   (b) 0.489
   (c) NO!
   (d) NO!

5) 0.45

6) (a) 0.0918
   (b) 0.6982